

INTERSECTION RULES FOR NON-EXTREME p -BRANESNobuyoshi Ohta¹*Department of Physics, Osaka University,
Toyonaka, Osaka 560, Japan***Abstract**

We give a model-independent derivation of general intersecting rules for non-extreme p -branes in arbitrary dimensions D . This is achieved by directly solving bosonic field equations for supergravity coupled to a dilaton and antisymmetric tensor fields with minimal ansätze. We compare the results with those in eleven-dimensional supergravity. Supersymmetry is recovered in the extreme limit if the backgrounds are taken to be independent. Consistency with non-supersymmetric solutions is also discussed. Finally the general formulae for the ADM mass, entropy and Hawking temperature are given.

¹e-mail address: ohta@phys.wani.osaka-u.ac.jp

There has been much progress in our understanding of classical solutions of supergravities in eleven and ten dimensions. These theories are the low-energy limits of the string theories and supposedly unifying M-theory of strings. Toroidally compactified, the solutions give rise to black hole solutions in lower dimensions, the study of whose quantum properties may be feasible in the framework of string theories. Also these solutions are known to play significant roles in strong coupling dynamics of string theories [1, 2]. It is thus important to better understand these classical p -brane solutions.

The single p -brane solutions have been discussed in refs. [3, 4, 5, 6, 7] for low-energy effective theories and in refs. [8, 9] for $D=11$ supergravity. It has then been noted that the more general solutions can be understood as intersecting ones of these fundamental p -branes [9]. A systematic approach to formulating the rules for the way how they can intersect has then been given in a number of papers [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. In particular, for $D = 11$ supergravity, Tseytlin stated the “harmonic superposition rules” for extreme M-brane solutions [10], and these rules have been generalized to non-extreme case in ref. [14]. Although their “rules” are consistent with most of the known solutions, it is not clear if there are any other solutions than those given by these rules. The questions we would like to ask here are how general these rules are and how severely they restrict the solutions for supergravities in $D = 11$ and lower dimensions.

Recently a general approach to these problems has been given in refs. [17, 18]. However, the authors discuss only the extreme case and make several ansätze. It would be quite interesting and important to extend this to the non-extreme case and also to clarify to what extent these ansätze are really ansätze but not something that may be derived from the field equations. This is the subject with which we are mainly concerned in this paper. In particular, we generalize this work in the following two respects. First, we deal with non-extreme case² by introducing an arbitrary function which characterizes non-extreme solutions and derive the rules from the general approach. Second, we show that most of the ansätze made in refs. [17, 18] are actually simple consequences of the field equations, thus clarifying the real assumption. We find that this extension is quite nontrivial. The field equations can be easily integrated and the consistency of the

²The extension to non-extreme case is also discussed in refs. [20, 19].

solutions reduces the problem of solving the field equations to an algebraic one.

The results of our analysis turn out to be consistent with the harmonic superposition rules in refs. [10, 14] and supersymmetry is recovered in the extreme limit if we require the independence of the background fields, even though we do not require unbroken supersymmetry. On the other hand, several non-supersymmetric solutions have also been discovered [19, 21]. We show that such solutions are allowed if we relax the condition imposed by the independence of the background fields. We also discuss the ADM mass, entropy and Hawking temperature of the resulting black hole solutions.

Let us start with the general action for gravity coupled to a dilaton ϕ and m different n_A -form field strengths:

$$I = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^m \frac{1}{2n_A!} e^{a_A\phi} F_{n_A}^2 \right]. \quad (1)$$

This action describes the bosonic part of $D = 11$ or $D = 10$ supergravities; we simply drop ϕ and put $a_A = 0$ and $n_A = 4$ for $D = 11$, whereas we set $a_A = -1$ for the NS-NS 3-form and $a_A = \frac{1}{2}(5 - n_A)$ for forms coming from the R-R sector.³ To describe more general supergravities in lower dimensions, we should include several scalars as in ref. [6], but for simplicity we disregard this complication in this paper.

From the action (1), one derives the field equations

$$\begin{aligned} R_{\mu\nu} &= \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + \sum_A \frac{1}{2n_A!} e^{a_A\phi} \left[n_A (F_{n_A}^2)_{\mu\nu} - \frac{n_A - 1}{D - 2} F_{n_A}^2 g_{\mu\nu} \right], \\ \square \phi &= \sum_A \frac{a_A}{2n_A!} e^{a_A\phi} F_{n_A}^2, \\ \partial_{\mu_1} \left(\sqrt{-g} e^{a_A\phi} F^{\mu_1 \dots \mu_{n_A}} \right) &= 0, \\ \partial_{[\mu} F_{\mu_1 \dots \mu_{n_A}]} &= 0. \end{aligned} \quad (2)$$

The last equations are the Bianchi identities.

We take the following metric for our system:

$$ds_D^2 = -e^{2u_0} f dt^2 + \sum_{\alpha=1}^p e^{2u_\alpha} dy_\alpha^2 + e^{2B} \left[f^{-1} dr^2 + r^2 d\Omega_{d+1}^2 \right], \quad (3)$$

³There may be Chern-Simons terms in the action, but they are irrelevant in our following solutions.

where $D = p + \tilde{d} + 3$, the coordinates $y_\alpha, (\alpha = 1, \dots, p)$ parametrize the p -dimensional compact directions and the remaining coordinates of the D -dimensional spacetime are the radius r and the angular coordinates on a $(\tilde{d}+1)$ -dimensional unit sphere, whose metric is $d\Omega_{\tilde{d}+1}^2$. The function f is introduced in order to describe the non-extreme solutions. Since we are interested in static spherically-symmetric solutions, all the functions appearing in the metrics as well as dilaton ϕ are assumed to depend only on the radius r of the transverse dimensions.

If the resulting metric has null isometry, say, in the direction y_1 , we can incorporate the boost charge by a well-defined step [22, 14]. Since this is quite straightforward, we simply concentrate on the diagonal metric (3).

For background field strengths, we take the most general ones consistent with the field equations and Bianchi identities. The background for an electrically charged q -brane is given by

$$F_{0\alpha_1\cdots\alpha_q r} = \epsilon_{\alpha_1\cdots\alpha_q} E', \quad (n_A = q + 2), \quad (4)$$

where $\alpha_1, \dots, \alpha_q$ stand for the compact dimensions. Here and in what follows, a prime denotes a derivative with respect to r .

The magnetic case is given by

$$F^{\alpha_{q+1}\cdots\alpha_p a_1\cdots a_{\tilde{d}+1}} = \frac{1}{\sqrt{-g}} e^{-a\phi} \epsilon^{\alpha_{q+1}\cdots\alpha_p a_1\cdots a_{\tilde{d}+1} r} \tilde{E}', \quad (n_A = D - q - 2) \quad (5)$$

where $a_1, \dots, a_{\tilde{d}+1}$ denote the angular coordinates of the $(\tilde{d}+1)$ -sphere. The functions E and \tilde{E} are again assumed to depend only on r .

The electric background (4) trivially satisfies the Bianchi identities but the field equations are nontrivial. On the other hand, the field equations are trivial but the Bianchi identities are nontrivial for the magnetic background (5).

We will solve the field eqs. (2) with the simplifying ansatz

$$\sum_{\alpha=0}^p u_\alpha + \tilde{d}B = 0. \quad (6)$$

This is the only assumption we make in solving the field eqs. (2). In particular, we do not make any ansätze on the background field E , in contrast to refs. [17, 18], but will show that the consistency of the field equations automatically determines the function.

The field equations (2) considerably simplify owing to the condition (6). For both cases of electric (4) and magnetic (5) backgrounds, we find that the field eqs. (2) are cast into

$$\left(u_0 + \frac{1}{2} \ln f\right)'' + \left(\frac{f'}{f} + \frac{\tilde{d}+1}{r}\right) \left(u_0 + \frac{1}{2} \ln f\right)' = \frac{1}{f} \sum_A \frac{D - q_A - 3}{2(D-2)} S_A (E_A')^2, \quad (7)$$

$$u_\alpha'' + \left(\frac{f'}{f} + \frac{\tilde{d}+1}{r}\right) u_\alpha' = \frac{1}{f} \sum_A \frac{\delta_A^{(\alpha)}}{2(D-2)} S_A (E_A')^2, \quad (\alpha = 1, \dots, p), \quad (8)$$

$$\begin{aligned} B'' + \sum_{\alpha=0}^p (u_\alpha')^2 + \frac{\tilde{d}+1}{r} B' + \tilde{d}(B')^2 + \frac{f'}{2f} \left(2u_0' + \frac{f'}{f} + \frac{\tilde{d}+1}{r}\right) + \frac{1}{2} (\ln f)'' \\ = -\frac{1}{2} (\phi')^2 + \frac{1}{f} \sum_A \frac{D - q_A - 3}{2(D-2)} S_A (E_A')^2, \end{aligned} \quad (9)$$

$$f \left[(B + \ln r)'' + \left(\frac{f'}{f} + \frac{\tilde{d}+1}{r}\right) (B + \ln r)' \right] - \frac{\tilde{d}}{r^2} = - \sum_A \frac{q_A + 1}{2(D-2)} S_A (E_A')^2, \quad (10)$$

$$r^{-(\tilde{d}+1)} \left(r^{\tilde{d}+1} f \phi' \right)' = - \sum_A \frac{\epsilon_A a_A}{2} S_A (E_A')^2, \quad (11)$$

$$\left(r^{\tilde{d}+1} S_A E_A' \right)' = 0, \quad (12)$$

where A denotes the kinds of q_A -branes and we have defined

$$S_A \equiv \exp \left(\epsilon_A a_A \phi - 2 \sum_{\alpha \in q_A} u_\alpha \right), \quad (13)$$

and

$$\delta_A^{(\alpha)} = \begin{cases} D - q_A - 3 \\ -(q_A + 1) \end{cases} \quad \text{for} \quad \begin{cases} y_\alpha \text{ belonging to } q_A\text{-brane and } \alpha = 0 \\ \text{otherwise} \end{cases}, \quad (14)$$

and $\epsilon_A = +1(-1)$ corresponds to electric (magnetic) backgrounds. For magnetic case we have dropped the tilde from E_A . Equations (7), (8), (9) and (10) are the $00, \alpha\alpha, rr$ and ab (angular coordinates) components of the Einstein equation in (2), respectively. The last one is the field equation for the field strengths of the electric backgrounds and/or Bianchi identity for the magnetic ones.

From eq. (12) one finds

$$r^{\tilde{d}+1} S_A E_A' = c_A, \quad (15)$$

where c_A is a constant. With the help of eq. (15), eq. (7) can be rewritten as

$$\left[r^{\tilde{d}+1} f \left(u_0 + \frac{1}{2} \ln f \right)' \right]' = \sum_A \frac{D - q_A - 3}{2(D-2)} c_A E_A', \quad (16)$$

which can be integrated to give

$$f \left(u_0 + \frac{1}{2} \ln f \right)' = \sum_A \frac{D - q_A - 3}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_0 \tilde{d}}{r^{\tilde{d}+1}}, \quad (17)$$

where c_0 is an integration constant. Similarly, we find that eqs. (8), (10) and (11) give

$$\begin{aligned} f u_\alpha' &= \sum_A \frac{\delta_A^{(\alpha)}}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\alpha \tilde{d}}{r^{\tilde{d}+1}}, \\ f (B + \ln r)' - \frac{1}{r} &= - \sum_A \frac{q_A + 1}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_b \tilde{d}}{r^{\tilde{d}+1}}, \\ f \phi' &= - \sum_A \frac{\epsilon_A a_A}{2} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\phi \tilde{d}}{r^{\tilde{d}+1}}, \end{aligned} \quad (18)$$

where c_α, c_b and c_ϕ are again integration constants.

Substituting eqs. (17) and (18) into (9) yields

$$\begin{aligned} &\left(\sum_A \frac{D - q_A - 3}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_0 \tilde{d}}{r^{\tilde{d}+1}} - \frac{1}{2} f' \right)^2 + \sum_{\alpha=1}^p \left(\sum_A \frac{\delta_A^{(\alpha)}}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\alpha \tilde{d}}{r^{\tilde{d}+1}} \right)^2 \\ &+ \tilde{d} \left(- \sum_A \frac{q_A + 1}{2(D - 2)} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_b \tilde{d}}{r^{\tilde{d}+1}} - \frac{f - 1}{r} \right)^2 + \frac{1}{2} \left(- \sum_A \frac{\epsilon_A a_A}{2} c_A \frac{E_A}{r^{\tilde{d}+1}} + \frac{c_\phi \tilde{d}}{r^{\tilde{d}+1}} \right)^2 \\ &+ f' \left(\sum_A \frac{c_A}{2} \frac{E_A}{r^{\tilde{d}+1}} + \frac{(c_0 - c_b) \tilde{d}}{r^{\tilde{d}+1}} + \frac{f - 1}{r} + \frac{\tilde{d} - 1}{2r} f \right) + \frac{1}{2} (f'' f - (f')^2) - f(f - 1) \frac{\tilde{d}}{r^2} \\ &= \frac{f}{2} \sum_A \frac{c_A}{r^{\tilde{d}+1}} E_A'. \end{aligned} \quad (19)$$

This equation must be valid for functions E_A of r .

From the E_A -independent part of eq. (19), one finds

$$f' - \frac{2c_0 \tilde{d}}{r^{\tilde{d}+1}} = c_\alpha = \frac{f - 1}{r} - \frac{c_b \tilde{d}}{r^{\tilde{d}+1}} = c_\phi = 0. \quad (20)$$

Remarkably all these constraints give a consistent solution

$$f = 1 - \frac{2\mu}{r^{\tilde{d}}}; \quad c_0 = \mu; \quad c_b \tilde{d} = -2\mu; \quad c_\alpha = c_\phi = 0. \quad (21)$$

We thus see that the non-extreme function f is determined by the consistency of the field equations and that the deformation parameter μ appears as an integration constant.

Using (21), we can rewrite eq. (19) as

$$\sum_{A,B} \left[M_{AB} \frac{c_A}{2} + r^{\tilde{d}+1} \left(\frac{f}{E_A} \right)' \delta_{AB} \right] \frac{c_B}{2} \frac{E_A E_B}{r^{2\tilde{d}+2}} = 0, \quad (22)$$

where

$$M_{AB} = \sum_{\alpha=0}^p \frac{\delta_A^{(\alpha)} \delta_B^{(\alpha)}}{(D-2)^2} + \tilde{d} \frac{(q_A+1)(q_B+1)}{(D-2)^2} + \frac{1}{2} \epsilon_A a_A \epsilon_B a_B. \quad (23)$$

Since M_{AB} is constant, eq. (22) cannot be satisfied for arbitrary functions $\frac{f}{E_A}$ of r unless the second term inside the square bracket is a constant. Requiring this to be a constant tells us that the function $\frac{f}{E_A}$ is harmonic or

$$E_A = N_A \frac{f}{H_A}; \quad (r^{\tilde{d}+1} H_A')' = 0, \quad (24)$$

where N_A is a normalization constant so that we can choose

$$H_A = 1 + \frac{Q_A}{r^{\tilde{d}}}. \quad (25)$$

In this way, the problem reduces to the algebraic equation (22) supplemented by (24) without making any assumption other than (6).

Equation (22) has two implications if we take independent functions for the background fields E_A . In this case, first putting $A = B$ in eq. (22), we learn that

$$\frac{c_A}{2} = \frac{\tilde{d} Q_A}{N_A M_{AA}} \equiv \frac{\tilde{d} Q_A}{N_A} \frac{D-2}{\Delta_A}, \quad (26)$$

where

$$\Delta_A = (q_A+1)(D-q_A-3) + \frac{1}{2} a_A^2 (D-2). \quad (27)$$

By use of eqs. (21), (24) and (26), eqs. (17) and (18) can be integrated with the results

$$\begin{aligned} u_0 &= - \sum_A \frac{D-q_A-3}{\Delta_A} \ln H_A, \\ u_\alpha &= - \sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln H_A, \\ B &= \sum_A \frac{q_A+1}{\Delta_A} \ln H_A, \\ \phi &= \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln H_A, \end{aligned} \quad (28)$$

where we have imposed the condition that the metric is asymptotically flat ($u_0, u_\alpha, B \rightarrow 0$ for $H_A \rightarrow 1$) and the dilaton vanishes to determine the final integration constants.

To fix the normalization N_A , we go back to eq. (13). Using (28), we find

$$S_A = H_A^2, \quad (29)$$

which, together with (15) and (26), leads to

$$N_A = \sqrt{\frac{2Q_A(D-2)}{(Q_A+2\mu)\Delta_A}}. \quad (30)$$

Note that (29) is not an ansatz but a result following from the field equations.

Our metric and background fields are thus finally given by

$$\begin{aligned} ds_D^2 &= \prod_A H_A^{2\frac{q_A+1}{\Delta_A}} \left[- \prod_A H_A^{-2\frac{D-2}{\Delta_A}} f dt^2 + \sum_{\alpha=1}^p \prod_A H_A^{-2\frac{\gamma_A^{(\alpha)}}{\Delta_A}} dy_\alpha^2 + f^{-1} dr^2 + r^2 d\Omega_{d+1}^2 \right], \\ E_A &= \sqrt{\frac{2Q_A(D-2)}{(Q_A+2\mu)\Delta_A}} \frac{f}{H_A}. \end{aligned} \quad (31)$$

where we have defined

$$\gamma_A^{(\alpha)} = \begin{cases} D-2 & \text{for } \begin{cases} y_\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases} \\ 0 & \end{cases}, \quad (32)$$

in agreement with the harmonic superposition rules for $D = 11$ supergravity [10, 14].

The second condition following from eqs. (22) is $M_{AB} = 0$ for $A \neq B$. As shown in ref. [17], this leads to the intersection rules for two branes: If q_A -brane and q_B -brane intersect over $\bar{q}(\leq q_A, q_B)$ dimensions, this gives

$$\bar{q} = \frac{(q_A+1)(q_B+1)}{D-2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B. \quad (33)$$

For eleven-dimensional supergravity, we have electric 2-branes, magnetic 5-branes and no dilaton $a_A = 0$. The rule (33) tells us that 2-brane can intersect with 2-brane on a point ($\bar{q} = 0$) and with 5-brane over a string ($\bar{q} = 1$), and 5-brane can intersect with 5-brane over 3-brane ($\bar{q} = 3$), again in agreement with refs. [9, 10]. Other implications of (33) for lower-dimensional supergravities are discussed in ref. [17].

In the above derivation, we have not imposed exact supersymmetry. Nevertheless, the results are consistent with supersymmetry, and if \mathcal{N} q -branes are involved in the solutions, there remain at least $1/2^{\mathcal{N}}$ supersymmetry in the extreme limit $\mu \rightarrow 0$. The

question then arises what happens to the non-supersymmetric solutions in refs. [19, 21]. As an example, take the solution in ref. [19]:

$$\begin{aligned} ds_{11}^2 &= H^{1/2} \left[H^{-3/2} (-f dt^2 + dy_1^2) + H^{-1/2} (dy_2^2 + dy_3^2 + dy_4^2) + f^{-1} dr^2 + r^2 d\Omega_5^2 \right], \\ F_{01\alpha r} &= E', (\alpha = 2, 3, 4); \quad E = \sqrt{\frac{Q}{2(Q+2\mu)}} \left(1 - \frac{Q+2\mu}{r^4+Q} \right), \end{aligned} \quad (34)$$

with $\tilde{d} = 4$.⁴ This solution may be interpreted as three intersecting 2-branes over a string. We see that here the same harmonic function is used for all the 2-branes, and hence the assumption of the independence of the background functions E_A is not satisfied in this solution. Indeed, it is easy to check that (34) is consistent with the condition (22) and (24). Thus the independence of the backgrounds imposes a strong constraint on the possible solutions.

Let us finally discuss general properties of the black hole solutions obtained from the above solutions. If we compactify the coordinates y_α , ($\alpha = 1, \dots, p$) on p -torus of common length L , they reduce to the $(\tilde{d}+3)$ -dimensional black holes with Einstein-frame metric

$$ds_{\tilde{d}+3}^2 = -G^{\tilde{d}}(r) f(r) dt^2 + G^{-1}(r) \left[f^{-1}(r) dr^2 + r^2 d\Omega_{\tilde{d}+1}^2 \right], \quad (35)$$

with

$$G(r) = \prod_A H_A^{-\frac{2(D-2)}{(\tilde{d}+1)\Delta_A}}. \quad (36)$$

We can read off the ADM mass from the asymptotic form of the metric g_{00} :

$$M_{ADM} = a \left[(\tilde{d}+1)\mu + (D-2)\tilde{d} \sum_A \frac{Q_A}{\Delta_A} \right], \quad (37)$$

where we have defined

$$a \equiv \frac{\omega_{\tilde{d}+1}}{\kappa^2} L^p, \quad \omega_{\tilde{d}+1} \equiv \frac{2\pi^{\frac{\tilde{d}}{2}+1}}{\Gamma\left(\frac{\tilde{d}}{2}+1\right)}, \quad (38)$$

and the D -dimensional Newton's constant is written as $G_D \equiv \frac{\kappa^2}{8\pi}$.

The $(D-2)$ -area and entropy are given by

$$\begin{aligned} A_{D-2} &= \omega_{\tilde{d}+1} L^p (2\mu)^{\frac{\tilde{d}+1}{\tilde{d}}} \prod_A \left(1 + \frac{Q_A}{2\mu} \right)^{\frac{D-2}{\Delta_A}}, \\ S_{D-2} &= \frac{2\pi A_{D-2}}{\kappa^2}. \end{aligned} \quad (39)$$

⁴The normalization and the constant part of E differ slightly from ref. [19] due to different conventions.

Near the extreme limit $\mu \sim 0$, the entropy behaves like

$$S_{D-2} \sim 2\pi a (2\mu)^\lambda \prod_A Q_A^{\frac{D-2}{\Delta_A}}, \quad (40)$$

where the constant λ is given by

$$\lambda \equiv \frac{\tilde{d}+1}{\tilde{d}} - \sum_A \frac{D-2}{\Delta_A}. \quad (41)$$

We see that this constant govern the behavior of the entropy in the extreme limit. It is worth noting that if we use a charge defined by

$$P_A \equiv \sqrt{Q_A(Q_A + 2\mu)}, \quad (42)$$

which reduces to Q_A in the extreme limit, the ADM mass can be written as

$$M_{ADM} = a\tilde{d} \left[(D-2) \sum_A \frac{\sqrt{P_A^2 + \mu^2}}{\Delta_A} + \lambda\mu \right]. \quad (43)$$

From the Euclideanized metric of (35), we find the Hawking temperature is given by

$$T_H = \frac{\tilde{d}}{4\pi(2\mu)^{1/\tilde{d}}} \prod_A \left(\frac{2\mu}{2\mu + Q_A} \right)^{\frac{D-2}{\Delta_A}}. \quad (44)$$

Near the extreme limit, this behaves like

$$T_H \sim \frac{\tilde{d}}{4\pi} (2\mu)^{1-\lambda} \prod_A Q_A^{-\frac{D-2}{\Delta_A}}. \quad (45)$$

Expressed in terms of the Hawking temperature, the entropy (39) becomes

$$S_{D-2} = a\tilde{d} \frac{\mu}{T_H}. \quad (46)$$

In the extreme limit, the Hawking temperature vanishes for $\lambda < 1$ whereas the entropy does for $\lambda > 0$. In particular, for the interesting case of $\lambda = 0$, the entropy (and the horizon area) is finite but the Hawking temperature vanishes.

To summarize, we have given quite a general model-independent derivation of the harmonic superposition rules in arbitrary dimensions. The only ansatz we make is the condition (6) and no other assumptions are necessary; all the others simply follow from the field

equations. Supersymmetry is recovered in the extreme limit if we require that the backgrounds be independent. If we do not stick to the latter condition, non-supersymmetric extreme solutions are also allowed. In all cases, the algebraic eq. (22) (together with (24)) must be satisfied, and this equation should be most useful to examine possible solutions. We have also discussed general formulae for entropy, area and Hawking temperature valid for all solutions. These should be useful for understanding the process of Hawking radiation of these black holes. We hope to discuss various properties of these solutions using the hints from dualities implied by underlying string dynamics elsewhere.

References

- [1] C. M. Hull and P. K. Townsend, Nucl. Phys. **B438** (1995) 109, hep-th/9410167.
- [2] E. Witten, Nucl. Phys. **B443** (1995) 85, hep-th/9503124.
- [3] A. Dabholkar, G. Gibbons, J. A. Harvey and F. Ruiz Ruiz, Nucl. Phys. **B340** (1990) 33.
- [4] G. T. Horowitz and A. Strominger, Nucl. Phys. **B360** (1991) 197.
- [5] M. J. Duff, H. Lü and C. N. Pope, Phys. Lett. **B382** (1996) 73, hep-th/9604052.
- [6] H. Lü, C. N. Pope, E. Sezgin and K. S. Stelle, Nucl. Phys. **B456** (1996) 669, hep-th/9508042; N. Khviengia, Z. Khviengia, H. Lü and C. N. Pope, Phys. Lett. **B388** (1996) 21, hep-th/9605077.
- [7] For reviews and earlier references, see M. Duff, R. Khuri and J. X. Lu, Phys. Rep. **259** (1995) 213, hep-th/9412184; K. S. Stelle, preprint, hep-th/9701088; H. Lü and C. N. Pope, preprint, hep-th/9702086.
- [8] R. Güven, Phys. Lett. **B276** (1992) 49.
- [9] G. Papadopoulos and P. Townsend, Phys. Lett. **B380** (1996) 273, hep-th/9603087.
- [10] A. A. Tseytlin, Nucl. Phys. **B475** (1996) 149, hep-th/9604035.

- [11] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B475** (1996) 164, hep-th/9604089.
- [12] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B475** (1996) 179, hep-th/9604166.
- [13] J. P. Gauntlett, D. A. Kastor and J. Traschen, Nucl. Phys. **B478** (1996) 544, hep-th/9604179.
- [14] M. Cvetič and A. A. Tseytlin, Nucl. Phys. **B478** (1996) 181, hep-th/9606033.
- [15] M. S. Costa, preprints, hep-th/9609181, hep-th/9610138.
- [16] A. A. Tseytlin, preprint, hep-th/9609212.
- [17] R. Argurio, F. Englert and L. Houart, preprint, hep-th/9701042.
- [18] I. Y. Aref'eva and O. A. Rytchkov, preprint, hep-th/9612236; I. Y. Aref'eva, K. S. Viswanathan, A. I. Volovich and I. V. Volovich, preprint, hep-th/9701092.
- [19] N. Ohta and T. Shimizu, preprint, hep-th/9701095.
- [20] I. Y. Aref'eva, M. G. Ivanov and I. V. Volovich, preprint, hep-th/9702079.
- [21] R. R. Khuri and T. Ortín, Phys. Lett. **B373** (1996) 56, hep-th/9512178.
- [22] D. Garfinkle, Phys. Rev. **D46** (1992) 4286.